Vermont's Transition to the Common Core State Standards:

Mathematics

Going Deeper into the Standards for Mathematics and Understanding Implications for Instruction

(Resources Embedded)

State of Vermont, Department of Education
August 2011

Forward: Professional Learning

The adoption of the *Common Core State Standards in English Language Arts and Mathematics* by the Vermont State Board of Education in August 2010 serves as a catalyst for the transformation of K-12 education in Vermont. Because the standards are anchored in the knowledge and skills for all students to be successful in college and career, the effectiveness of their implementation requires all educators to teach in a manner consistent with the intended purpose of common rigorous standards. This expectation, in turn, will require sustained professional development (PD) efforts in all Vermont schools during the next four years.

This transition period between the adoption of the Common Core State Standards (CCSS) 2010 and the first administration of the assessment of the CCSS in 2015 requires a phased approach for Vermont schools and districts; with successive levels of implementation, each a prerequisite for the next phase.

- **Phase 1** consists of building awareness of the CCSS among educators, including familiarity with the rationale for having common standards across states.
- **Phase 2** requires going deeper into the standards to identify, understand and implement significant instructional shifts implicit in the mathematics and English Language Arts (ELA) standards.
- **Phases 3 and 4** will focus on curriculum adoption and accessing the full range of assessment strategies to ensure success for all students.

The Implementation Timeline SY2011-12 and the Professional Learning Transition Timeline provide graphic representations of the Transition Phases, particularly Phase 2.

Each of the phases demands intensive professional development at the local level.

Research has shown that successful professional development means "a comprehensive, sustained, and intensive approach to improving teachers' and principals' effectiveness in raising student achievement," *Learning Forward* (formerly the National Staff Development Council).

Learning Forward's Standards for Professional Learning below outline characteristics of professional learning that lead to effective teaching practices, supportive leadership, and improved student results:

Learning Communities: Professional learning that increases educator effectiveness and results for all students occurs within learning communities committed to continuous improvement, collective responsibility, and goal alignment.

Leadership: Professional learning that increases educator effectiveness and results for all students requires skillful leaders who develop capacity, advocates, and create support systems for professional learning.

Resources: Professional learning that increases educator effectiveness and results for all students requires prioritizing, monitoring, and coordinating resources for educator learning.

Data: Professional learning that increases educator effectiveness and results for all students uses a variety of sources and types of student, educator, and system data to plan, assess, and evaluate professional learning.

Learning Designs: Professional learning that increases educator effectiveness and results for all students integrates theories, research, and models of human learning to achieve its intended outcomes.

Implementation: Professional learning that increases educator effectiveness and results for all students applies research on change and sustains support for implementation of professional learning for long term change.

Outcomes: Professional learning that increases educator effectiveness and results for all students aligns its outcomes with educator performance and student curriculum standards.

Educators in schools and districts across Vermont will need systems that incorporate these research-based elements of practice that work together to create a consistent culture of learning.

The Common Core State Standards powered by effective professional development systems are a significant driver of the transformation of education in Vermont. A truly effective implementation of the CCSS demands innovation in learning environments, technology and systems that support all students to meet rigorous 21st century expectations. This document serves as a guide for schools and districts

in their implementation of the CCSS within the broader frame of transforming opportunities for all students. It will evolve and grow as new resources are created or identified and further connections are mapped to a new course for education in Vermont.

Vermont's Transition to the Common Core State Standards: *Mathematics*

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Introduction: The *Common Core State Standards* in Mathematics

This is an exciting time for Vermont mathematics educators. With the adoption of the Common Core State Standards (CCSS), Vermont will build on work of the last two decades and move towards preparing Vermont students to be college and career ready. The CCSS are in line with the expectations of high performing countries and were designed to attend to the "mile wide and inch deep" curriculum typical in US standards. One immediate change Vermont schools will notice is the decrease in mathematical topics at grades K-5. An effect of the narrowed focus of mathematical topics, elementary students will spend more time on essential mathematics at a deeper level ensuring a solid foundation for future mathematical study and being college and career ready. While elementary teachers will have fewer mathematical topics to teach in their curricula, they will be challenged to work at a much deeper level than our current GEs require. In this process of "teaching more of less" in the K-5 curriculum, some mathematical topics will now be taught in later grades and a few eliminated all together.

It is important to recognize that "fewer standards" are no substitute for focused standards. Achieving "fewer standards" would be easy to do by resorting to broad, general statements. Instead, these Standards aim for clarity and specificity.

CCSS

The CCSS also were written with the idea of coherence. The standards were built on learning progressions. The standards explicitly build students' content knowledge from the previous years' learning. For example, the concepts and skills in the K-5 domain of Operations and Algebraic Thinking flow into the middle grades Expressions and Equations domain. There are intentional and explicit connections built between each grade's learning, paving the way to college and career readiness.

The Vermont Department of Education (DOE) formed a mathematics advisory committee that met several times in the spring of 2011. The committee, consisting of state assessment coordinators, professional development providers, curriculum coordinators, and VPDN teacher leaders, met to address the following questions in regards to support for the implementation of the CCSS.

1) What professional development do teachers need in order to effectively implement the Mathematics CCSS?

2) What delivery systems could support this professional development most effectively?

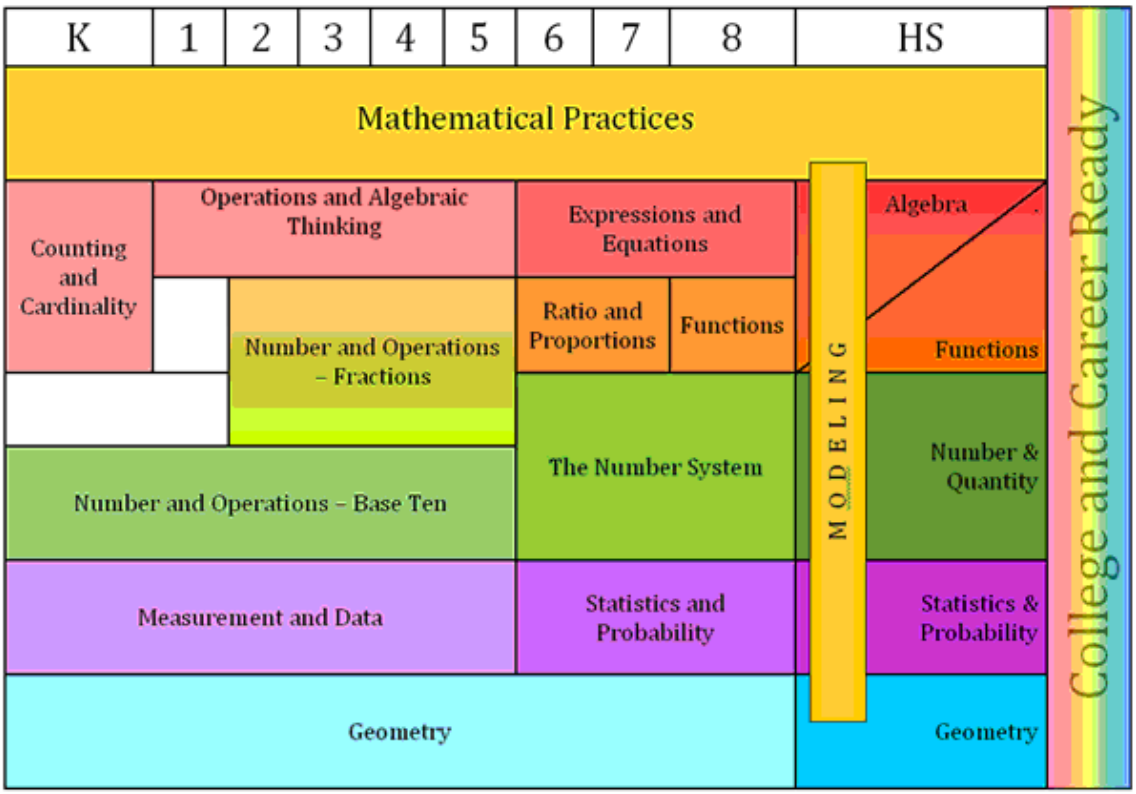

Figure 1: CCSS K-12 Mathematics Progression of Domains

To answer these questions the group first identified content areas of the CCSS in mathematics that would reflect a significant change in mathematics curriculum, instruction, and assessment. The following features were identified.

- The Standards for Mathematical Practices are standards that require a different definition of teaching, learning, and mathematical proficiency for all grades.

- The CCSS in mathematics were designed based on current understanding of learning trajectories and mathematical coherence.
  Example 1: Algebra at grades K-5 is NOT a separate domain; rather it is integrated into the development of number and operation concepts.
  Example 2: Measurement is used as a spring board to fractions as quantities that ultimately supports the understanding of variables.

- The development of number concept is the major focus at grades K-5. To accomplish this, there are two notable changes: some topics typically taught

at grades K-5 are not introduced until later grades (e.g., probability, statistical analysis) and other topics are eliminated. These changes will allow students more time to learn the foundations of mathematics with more depth.

What Are the Instructional Priorities for Successful Implementation of the CCSS?

Recommendations

- K-12 Focus: Mathematical Practices
- Elementary Focus: Operations and Algebraic Thinking
- Middle Level Focus: Ratio and Proportions
- Middle Level Focus: Expressions, Equations and Functions
- High School Focus: Modeling

Each Instructional Priority has three essential elements:

- A rationale for the selection of this priority.
- Specific recommendations for professional development that are consistent with the standards identified by Learning Forward.
- A range of vetted resources and references for schools/districts to use with educators at all grade levels.

Vermont's Transition to the CCSS in Mathematics lays out in detail the instructional priorities that are the most significant and that will take both time and effort to fully implement in Vermont classrooms. Captions also show areas where topics can be eliminated and where important ideas are highlighted. However, both educators and students will benefit -- in the short and long term -- from the rigorous instruction embedded in this professional learning. There is important work to be done, and we urge curriculum and teacher leaders to review this document carefully and make thoughtful choices for the necessary transition in their schools.

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K-12 Focus: Mathematical Practices

The Common Core State Standards (CCSS) will further the work of Vermont educators by raising our goal to a clear definition of college and career readiness. The Standards for Mathematical Practices describe the "processes and proficiencies" that ensure students become mathematicians and thinkers. The growth and development of the eight Mathematical Practices must be supported at all grade levels. A close examination of each of the Mathematical Practices is important in developing shared understanding and dispelling underlying misconceptions students may have. However the implementation of the Common Core is most powerful as an amalgamation of several (practice and/or content) standards.

The Mathematical Practices are enhanced when teachers have:

- A deep understanding of the practices and how they relate to each other and support mathematical proficiency
- An understanding of how mathematical practices and mathematical topics interact and how they can be integrated into instruction and assessment in increasing the rigor of the content
- An understanding of how and why mathematical practices must be integrated into instruction and assessment

Common Core Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Recommendation for Professional Development

It is the recommendation of the committee that all professional development for the Standards of Mathematical Practices should:

1. Develop an awareness of the Standards of Mathematical Practices
2. Model Standards of Mathematical Practices during all PD opportunities always integrating into mathematical topics
3. Provide opportunities for teachers to construct an understanding and deepen their own knowledge and experience of the Standards of Mathematical Practices through multiple mathematical domains
4. Provide collaborative opportunities for teachers to reflect and improve their practices in integrating the Standards of Mathematical Practices in instruction of all mathematical content
5. Define and design ways to collect evidence to ensure that students are meeting these standards

"The artificial separation of arithmetic and algebra deprives the students of powerful ways of thinking about mathematics in the early grades and makes it more difficult for them to learn algebra in the later grades. Understanding takes a long time to develop."

Carpenter, Franke and Levi, 2003

Resources and Essential Processes:

These resources are intentionally sequenced to be used in an on going, embedded, professional development process.

<u>Recommendation #1:</u>

- Introduction to the CCSS for All Grades
- Using the following questions as a guide, read through the Mathematical Practices closely:
  - What parts of this document are familiar to your classroom instruction? Support with examples.
  - What in this document is different from your classroom instruction?
- Examination of Mathematical Practice "Look Fors" from Jonathan Wray https://sites.google.com/site/commoncoreinvermont/file-cabinet/math-practices-look-fors

Recommendation #2:

- Diving Deeper into the Common Core Standards for Mathematics: Leading with the Mathematical Practices -A webinar that introduces Mathematical Practices. http://ncsmonline.org/docs/events/webinars/NCSMCCSSWebinar2011-02-23Presentation.pdf
- Effective Questions Posters: These PBS TeacherLine posters give some questions that administrators can build into their professional learning opportunities with teachers that model the practices that the Common Core State Standards value and expect. https://docs.google.com/viewer?a=v&pid=explorer&chrome=true&srcid=0ByiW_dRvIJEoMmI3NTk1NDgtYTEyNS00ZDBlLTgxOTktYzY2OGU1ZDkxNDMw&hl=en

Recommendation #3:

- What is a Mathematically Proficient Student? Using a copy of the Common Core Mathematical Practices; engage in the following Think, Pair, and Share activity.
  Individually, circle the verb in each of the Practices. What strikes you as important? For example, to what extent does your school/district promote students proficiency on practice #2. Identify your response on a scale of 1 to 6 with 1 being Low and 6 as the Highest what is your evidence? With a partner, discuss and come to consensus. Share with the larger group. (Adapted from work by Diane J. Briars, NCSM President)
- **Explore RIGOR by Connecting the Standards for Mathematical Practice to the Content Standards:** Participants examine content standards to see how they connect with the Standards for Mathematical Practice and how in tandem they form the basis of a rigorous curriculum. (Appropriate for Pre-K-12.) http://www.doe.mass.edu/candi/commoncore/mathexplore/

Recommendation #4:

- NCSM Illustrating Mathematical Practices
  These ready-to-use PD materials are designed to help teachers understand the Standards for Mathematical Practice and implement them in their classrooms. Each module supports a 1.5- to 3-hour session that focuses on one or two mathematical practices. http://www.ncsmonline.org/ccss/materials.html

- Use current brain research to change the way your students think about mathematics. Students will become mathematically proficient more easily if they are thinking positively about learning mathematics. This topic is addressed in a recent ASCD publication: Willis, Judy *Learning to Love Math;*

- *Chapter 1 Reversing Math Negativity with an Attitude Makeover.* ASCD 2010. The introduction and Chapter 1 can be read by at this site. http://www.ascd.org/publications/books/108073.aspx

Recommendation #4

- Examination of Mathematical Practice "Look Fors" from Jonathan Wray https://sites.google.com/site/commoncoreinvermont/file-cabinet/math-practices-look-fors

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Elementary Focus: Operations and Algebraic Thinking

The path to college and career readiness starts in kindergarten. The strength of a student's foundational knowledge of mathematics is crucial in order to ensure future success. Perhaps the biggest shift for K-5 teachers as they move their practice to match the CCSS is the way in which algebraic thinking is embedded into the area of operations with numbers. That is, students develop their fluency of operations through their understanding of place value, properties, and relationships. In this way algebra can be thought of as generalized arithmetic. In turn these understandings will form the foundation as students move to more formal algebraic thinking. It is a far deeper way of becoming operationally fluent than we have previously encouraged.

Classroom instruction that supports children's algebraic thinking is marked by rich conversation in which children make and explore mathematical conjectures, build arguments to establish or refute these conjectures, and treat established conjectures (generalizations) as important pieces of shared classroom knowledge.

Blanton, 2008

Operations and Algebraic thinking is enhanced by:

- The content knowledge of the teacher. The teacher understands the mathematics enough to introduce multiple contexts and examples of the mathematics at hand. S/he also understand how operational thinking is an essential building block to understanding of fractions, algebra and other more advanced studies of mathematics.
- Opportunities for students to make and prove conjectures, reason abstractly and quantitatively, leading to generalizations about the properties of numbers and operations.
- A focus on the significance and importance of equality as a relationship between two equal quantities.
- Continued and repeated attention to the concepts of algebra as new operations, number systems and problem types are introduced.

Recommendation for Professional Development

The first wave of PD K-5 should explicitly focus on how algebraic thinking can be developed through the use of conjecture and proof, understanding equality, and how the properties of operations support the development of operational fluency.

Teachers can begin to phase out these mathematical topics as a means to teach remaining mathematical topics at a greater depth.

The following topics are currently assessed in NECAP and absent in the CCSS:
K-1: Non-numeric patterns (AB patterns)
K-1: Money (Coins)
K-4: Three-dimensional Geometry
K-5: Transformations
K-6: Probability

1) Focus on the domain of operations and algebraic thinking with particular attention to additive, multiplicative, and fractional reasoning
2) Focus on the domain of operations and algebraic thinking with particular attention to equality
3) Focus on the domain of operations and algebraic thinking with particular attention to properties of operations
4) Focus on the domain of operations and algebraic thinking with particular attention to strategies for fact fluency that use understanding of equality, and the properties of operations
5) Engage teachers in conjectures, justifications and proof that lead to strengthening their curriculums

Resources and Essential Processes:

These resources are intentionally sequenced to be used in an on going, embedded, professional development process.

Recommendation #1:

The Primary Numbers and Operations (PNOA) assessments were created by Vermont educators with the support of the Vermont DOE. It is an interview style assessment for K-2 students. It's located in Riverdeep Learning Village http://rlv.education.vermont.gov/lv/admin/login.jsp

Recommendation #2:

"Equality: Ways to Help Your Students Understand the Equal Sign" presented at a NCCTM conference in 2009. Understanding the equal sign is a critical skill that students must have in order to avoid many misconceptions as they learn mathematics. Here is a resource,
http://teacheracademy.org/docs/Math_equality_09_doc.pdf

Carpenter, Franke, and Levi's book, *Thinking Mathematically (2003)* is an easy read for teachers. Chapter 2 Equality is a downloadable free sample from the publisher.
http://www.heinemann.com/shared/onlineresources/E00565/chapter2.pdf

Recommendation #3:

The four number operations are governed by the commutative, associative and distributive properties. These properties should be used to understand *how and why* of computation. The Department of Education and Early Childhood Development site for Victoria, Australia provides some background and teaching strategies for these properties.
http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/structure/st275ma.htm

Recommendation #4:

It is imperative that students develop fact fluency in the elementary grades so they are not trying to solve 7+8 at the same time they are working on higher level mathematics. Fluency is achieved by practice and strategic conceptual practice, not memorization without understanding.
Parrish, Sherry. 2010. *Number Talks: Helping Children Build Mental Math and Computation Strategies.* Sausalito, CA. Math Solutions.

Recommendation #5:

Carpenter, Franke, and Levi's book, *Thinking Mathematically (2003)* is a 146-page book which provides excellent information on conjecture, justification and proofs. These topics are the focus of Chapter 4 -7. *Thinking Mathematically: Integrating Arithmetic & Algebra in Elementary School*. Portsmouth, NH: Heinemann.
Maria Blanton's book, *Algebra and the Elementary Classroom; Transforming Thinking, Transforming Practice* (2008). Chapter 6 focuses on teaching practices that develop algebraic thinking skills in their students. Chapter 6 is available online as the free sample of the book at
http://www.heinemann.com/shared/onlineresources%5CE00946%5Cblanton00946Sample.pdf

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Middle Level Focus: Ratio and Proportions

Proportion is a mathematical thread that binds many concepts in algebra, geometry and statistics. Despite its importance, this is a mathematical topic that many students have difficulty in understanding its applicability to a variety of contexts. In many modeling problems, it is necessary to identify a proportional relationship that arises in a real-world situation and analyze or extend that relationship.

Ratio and Proportion is enhanced by:

- Students engaged in a range of problem situations across the content domains at Middle Level (ML).
- Instruction that includes both proportional and non-proportional situations to mathematically discriminate between them.
- Instruction that focuses on the mathematical structure (multiplicative scaling up and down) of proportionality as opposed to procedures to solve proportions that lead to efficient and generalizable strategies.

Standards calling for understanding rather than solving a specific type of problem ... Teachers should be encouraged to emphasize problems in which students explore ratios and rates in real-world contexts.

B. McCallum

Recommendation for Professional Development

It is the recommendation of the committee that all professional development for the Ratio and Proportion should have teachers:

1) Understand how multiplicative reasoning and fractional understanding is foundational for the development of proportionality.

2) Understand how students develop proportional thinking including pre-conceptions and misconceptions that may interfere with learning new concepts or solving related problems.

3) Understand how proportionality spans the domains by trying out and analyzing evidence in students' work that engage the students in a range of mathematical practices (problems solving, modeling and structures).

Resources and Essential Processes:

These resources are intentionally sequenced to be used in an on going, embedded, professional development process.

Recommendation #1:

The National Council of Teachers of Mathematics (NCTM) Book written by Bonnie Litwiller, *Making Sense of Fractions, Ratios, and Proportions: 2002 Yearbook,* is rich with research, activities that teachers can utilize as well as student responses to analyze and better understand how fractions relate to proportions.

Recommendation #2:

The National Research Council's book, *How Students Learn: Mathematics,* they speak about the importance to address student misconceptions and preconceptions in Chapter 5: Mathematical Understanding.

The Vermont Mathematics Partnership's Ongoing Assessment Project (OGAP) is a cognitively-based formative assessment instructional intervention in mathematics. OGAP focuses on fractions, multiplicative reasoning and proportional reasoning in grades 2-8. It includes materials related to cognitive research, strategies, and tools. http://www.vermontinstitutes.org/index.php/vmp/ogap

Recommendation #3:

Inside Mathematics is a professional resource for educators that feature classroom examples of innovative teaching methods and insights into student learning, tools for mathematics instruction that teachers can use immediately, and video tours of the ideas and materials on the website. Inside Mathematics will be aligning its tools and examples to the Common Core. www.insidemathematics.org/

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Middle Level Focus: Expressions, Equations and Functions

The Expression and Equations domain and the Functions domain in grade 8 build upon work at elementary grades in Operations and Algebraic Thinking. It provides an important bridge to algebra and functions at the high school level. Beginning to formalize more mathematical patterns in a variety of contexts is an area of emphasis, so teachers will need to extend the situations in which they have students engage with mathematics in this important work.

Expressions, Equations, and Functions instruction is enhanced by:

- An understanding on how expressions, equations, and functions build on the elementary work on operations and algebraic thinking.
- Opportunities for students to make conjectures, proofs, and generalize mathematical ideas.
- Attention to understanding the quantities, and relationships between the quantities specifically when interpreting graphical representations of a function.
- An understanding of how expressions, equations and functions support mathematical modeling.

Recommendation for Professional Development

It is the recommendation of the committee that all professional development for the Expressions, Equations and Functions should:

1) Build understanding of how the operations and algebraic thinking in the elementary domain prepares students for this work.
2) Provide opportunities for teachers to introduce conjectures, proofs, justifications and generalizations into their classrooms.
3) Build understanding of how quantities relate to each other in defining and generalizing functions.
4) Build understanding of the math education research about equality and functions.

Resources and Essential Processes:

These resources are intentionally sequenced to be used in an on-going, embedded, professional development process.

Recommendation #1:

Teachers should read the K-5 standards to understand the progression of learning. The illustrative mathematics website allows you view the content standards by domain or grade level.
http://illustrativemathematics.org/standards/k8

The Progressions project's document on the domain of Operations and Algebraic Thinking Grades K-2 show the basic elements (Grades 3-5 should be forthcoming)
http://commoncoretools.files.wordpress.com/2011/05/ccss_progression_cc_oa_k2_2001_05_08.pdf

CPRE's Learning Trajectories in Mathematics pages 41-53 is Phil Daro's section of this informative publication which talks about the importance of learning trajectories in the development of the standards.
http://ncsmonline.org/docs/ccss/learning%20trajectories%20in%20math_cpre%20report.pdf

Recommendation #2:

The book *Conjecture and Proof* by Miklos Laczkovich is an instructional book which shows a different way of thinking about proof and mathematics.

The NCTM's Principals and Standards for School Mathematics has a chapter on reasoning and proof.
http://www.fayar.net/east/teacher.web/math/Standards/document/chapter3/reas.htm#bp1

Recommendation #4:

The Progressions project's document on the domain of Expression and Equations are valuable reading in understanding how the development of operations and equality build the foundation for further mathematical study.
http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_ee_2011_04_25.pdf

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High School Focus: Modeling

The Common Core State Standards were designed to bring a greater focus and coherence to a student's study of mathematics while ensuring to bridge our current gap to College and Career Readiness. Students must have access to a rigorous curriculum which ensures they work to transfer their mathematical understanding to new situations that simulate the messiness of real world problems. Mathematically proficient students look at problems outside of mathematics, describe or interrupt that problem using mathematics, come to a mathematical solution and then interrupt that solution back to the context of the real problem. It is one of the eight Mathematical Standards but also a Conceptual Category in the High School standards to ensure it is emphasized.

The modeling process is enhanced by:

a. Richness of the problem to invite open-ended investigation. Problems must invite a variety of viable answers and multiple ways to represent and solve.
b. Context of the problem. Selecting real-world problems is important, and real-world problems should tap into student past and future experiences and interest.
c. Teacher and student understanding of the modeling process. Teachers and students who have had prior experience understand the modeling process better and seek ways to incorporating modeling in classrooms to enhance the learning of mathematical content.
d. Teacher and student understanding of the context. Background information & experience is needed and gained through a variety of resources.

When engaged in the modeling process, modelers go through iterations of expressing, testing, and revising the trial model. In so doing, they simultaneously improve their model and also develop deeper understandings of the constraints and limitations that still exist at each stage of model development, and learn to articulate (to group members) the trade-offs and benefits of a particular model.

J. Zawojewski

Recommendation for Professional Development

It is the recommendation of the committee that all professional development for Modeling should:

1) Require teachers to identify and read all the modeling standards embedded in each of the high school conceptual categories.

2) Provide support in helping teachers begin to embed modeling situations into their current curricula.
3) Provide the opportunity to share modeling practices and items with colleagues to build knowledge of both the modeling process and the necessary content for modeling tasks.

Resources and Essential Processes:

These resources are intentionally sequenced to be used in an on going, embedded, professional development process.

Recommendation #1:

The Vermont DOE created High School Common Core Scavenger Hunt will bring awareness for all educators as well as work to located the different modeling standards in each conceptual category. https://sites.google.com/site/commoncoreinvermont/home/mathematics-high-school-scavenger-hunt

The California Math Project has created a warehouse of resources, assessments, etc. that focus on high school modeling. http://caccssm.cmpso.org/high-school-modeling-task-force/high-school-modeling-resources

Recommendation #2:

The Shell Centre was charged with creating some examples of problems that incorporate the Mathematical Practices. This website gives teachers some examples of mathematics problems that link to the Mathematical Practices. http://map.mathshell.org.uk/materials/stds.php

Recommendation #3:

Inside Mathematics is a professional resource for educators that features classroom examples of innovative teaching methods and insights into student learning, tools for mathematics instruction that teachers can use immediately, and video tours of the ideas and materials on the website. Inside Mathematics will be aligning its tools and examples to the Common Core. www.insidemathematics.org/

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Vermont Department of Education

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Julie Conrad joined the Department of Education (DOE) in 2010 after teaching for high school mathematics for the last ten years at Colchester High School. She is currently the Middle School and High School Mathematics Assessment Coordinator for the State of Vermont. She was a key member of the Vermont's Higher Education Collaboration (HEC) Teaching All Secondary Students (TASS) development and instructional team working to create high school renewal. She was an independent educational consultant working with schools in Vermont and throughout the Northeast for five years before assuming her position at the state department.

Kathy Renfrew kathy.renfrew@state.vt.us

Kathy Renfrew is in her fourth year as the Elementary Math and Science Assessment Coordinator for the State of Vermont DOE. As part of her work at the DOE, she is very involved with the transition to the Common Core Standards in Mathematics and English Language Arts. She is a Nationally Board Certified teacher with 30 years of experience in the classroom and a Presidential Awardee in Science. She currently serves on the board of Vermont Science Teachers Association, a committee member in National Science Teachers Association (NSTA), and as an online advisor at the NSTA Learning Center.

Vermont CCSS Mathematics Advisory Team

Mary Abele-Austin currently works at Thatcher Brook Primary School and Crossett Brook Middle School as the math coach / coordinator. This is her 8th year in this capacity. Mary is actively implementing the lesson study model of professional development with her math teachers. She is supported in this work through a math leader class in Montpelier (2nd yr.) Mary is currently a math network leader (8 yrs.) with the DOE and has led a number of workshops for teachers around the state in mathematics instruction. Before becoming a math coach, Mary worked as a 3rd/4th grade classroom teacher. Mary graduated from the University of Vermont in 1989 with a BS in Elementary Education. She completed her Master's Degree for UVM and the Vermont Mathematics Initiative (VMI) in May of 2010.

Betsy Allen currently serves as a Math Teacher Leader K-6 at Hardwick Elementary School after having taught for 23 years. She loves working with teachers, look deeply at how kids learn math, what are effective instructional strategies, and designing intervention programs for students who need additional practice. She is a Math Network Leader and was a part of the team of teachers that with Loree Silvis' guidance developed the Primary Number and Operations (PNOA) K-2 Assessments. She graduated from the VMI program in 2003.

Michael Caraco is currently the Math Department Chair at Burr & Burton Academy (BBA). Prior to BBA, he worked as the Associate Director of Bennington College's Center for Creative Teaching graduate program, math department chair at Mt. Greylock in Williamstown, MA, and as a math teacher at King & Queen Central High School in King & Queen Courthouse, VA. His areas of interest have included assessment, curriculum, technology, and math competitions.

Michelle Harper is a certified Vermont educator for Grades 5-9 Math and Science, but currently teaches only math at Williamstown Middle High School. Her focus has been 7th and 8th grade, teaching Connected Math 2, but she also teaches Algebra 1 to advanced math learners in the middle school and remedial numeracy and fractions to struggling 7th graders. She has been on the VT DOE Math Network Leadership team for one year. She is also being mentored by Mahesh Sharma through her school district working on math readiness skills and how students learn math sequentially, as well as developing vertical alignment and common assessments with her district math team.

Beth Hulbert is the mathematics coordinator for Barre Supervisory Union in VT where her responsibilities include implementation and oversight of mathematics curriculum, modeling effective instruction, facilitating the development of local mathematics assessments, and professional development. As part of a Math Science Partnership (MSP) grant she designed and implemented a comprehensive mathematics intervention/professional development model that has been replicated by dozens of schools across the country. She has also been on the development team for the OGAP which is a research-based formative assessment and professional development program in the key content areas of multiplicative reasoning, fractional reasoning and proportional reasoning.

Marge Petit is an independent educational consultant focusing on mathematics instruction and assessment. Currently, Marge's primary work is supporting the development and implementation of the Vermont Mathematics Partnership OGAP formative assessment project, providing instructional support, and supporting mathematics teacher leaders. Marge Petit brings to this work nearly two decades of research and development in standards-based restructuring efforts at the classroom, district, state and national levels, and 23 years of experience as a classroom teacher primarily in mathematics and science. Her experience in the implementation of a standards-based system extends from the classroom, to research and development, to state and national level policy.

Loree Silvis is an experienced primary-grade teacher who left the classroom several years ago to delve deeply into the cognitive research on how young children construct understanding of important foundational mathematical concepts. In support of early educators, she led the development of two graduate level courses which focus on number and additive reasoning and facilitates these courses throughout Vermont. She recently co-led the development of the PNOA with support from the Vermont DOE and is currently

consulting with several districts in the development of comprehensive assessment systems in the primary and elementary levels.

Sandi Stanhope was a classroom teacher for 26 years, teaching grades 1 & 2. During that time she started to do in-depth reading of the research focused on early numeracy and additive reasoning to make more of a positive impact on her instruction. In the past seven years, she was a mathematics interventionist for grades K-6 and then a mathematics coach for teachers in grades K-8, all at St. Albans City School. She has provided professional development in central and northwestern Vermont. In 2010, she completed VMI and became part of the instructional and mentoring staff which allows me to share my work around Early Numeracy and Additive Reasoning.

Jean Ward was most recently the Director of Curriculum, Assessment and Professional Development for the Bennington-Rutland Supervisory Union (BRSU). Jean specializes in science and math PD for K-6/8 teachers. She is currently involved in OGAP training and promotion of algebraic thinking in math. She helps to administer and teaches for the Southwest Vermont MSP Grant which focuses on science content, writing, assessment, and technology. Jean finalized her work with the BRSU in June and now enters the world of consulting and semi-retirement.

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References

Briars, D. J., Mills, V., & Mitchell, S. (n.d.). *Diving deeper into the common core standards for mathematics: Leading with the mathematical practices* [webinar with PowerPoint]. Retrieved August 9, 2011, from National council of supervisors' mathematics website: http://ncsmonline.org/docs/events/webinars/NCSMCCSSWebinar2011-02-23Presentation.pdf

California Math Project. (2011). *High School Modeling Resources.* Retrieved, on August 17, 2011, from the California Math Project website: http://caccssm.cmpso.org/high-school-modeling-task-force/high-school-modeling-resources

Carpenter, T. P., Franke, M. L., & Levi, L. (2003). *Thinking Mathematically: Integrating Arithmetic & Algebra in Elementary School*. Portsmouth NH: Heinemann.

Common core state standards for mathematics. (2010). Retrieved August 17, 2011, from the Common core state standards initiative website: http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

Consortium for Policy Research in Education. (2011). *Learning Trajectories in Mathematics.* Philadelphia, PA: CPRE

Kelley, K. (2009, October). *Equality: Ways to help your students understand the equal sign*. Conference Workshop presented at NCCTM 39th Annual State Conference.

Laczkovich, M., (2001). *Conjecture and Proof.* Cambridge, MA: The Mathematical Association of America

Litwiller, B., (2002). *Making Sense of Fractions, Rations, and Proportions: 2002 Yearbook.* Reston, VA: NCTM.

McCallum, W., *Tools for Common Core Standards.* Retrieved on August 17, 2011, from Dr. William McCallum's web blog: http://commoncoretools.wordpress.com/

Massachusetts Department of Education. (n.d.) *Explore RIGOR by connecting the standards for mathematical practice to the content standards.* Retrieved August 17, 2011, from: http://www.doe.mass.edu/candi/commoncore/mathexplore

Mathematics Developmental Continuum P-10 – Structure [Properties of Operations]. (2006, July 21). Retrieved August/September 10, 2011, from Department of Early Childhood and Education State of Victoria, Australia website: http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/structure/st275ma.htm

National Research Council. (2005). *How Students Learn: Mathematics in the Classroom*. Edited by M. S. Donovan and J. D. Bransford. Washington, D.C.: The National Academies Press

NCSM illustrating math practices [Power points, Documents]. (n.d.). Retrieved August 9, 2011, from National council supervisors' mathematics website: http://www.ncsmonline.org/ccss/materials.html

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics.* Retrieved, on August 17, 2011, from the NCTM website: http://www.nctm.org/standards/content.aspx?id=16909

North Carolina Teacher Academy (n.d.) *Equality: ways to help your students understand the equal sign.* Retrieved August 17, 2011, from: http://teacheracademy.org/docs/Math_equality_09_doc.pdf

The Noyce Foundation, inside mathematics. (n.d.). *Common core standards for mathematical practices.* Retrieved August 17, 2011, from: http://www.insidemathematics.org/index.php/commmon-core-math-intro

Parrish, S. (2010). *Number Talks: Helping Children Build mental Math and Computation Strategies*. Sausalito, CA: Math Solutions.

Public Broadcasting Station. (n.d.) *Developing mathematical thinking with effective questions.* Retrieved August 17, 2011, from: https://docs.google.com/viewer?a=v&pid=explorer&chrome=true&srcid=0ByiW_dddRvIJEoM3NTk1NDgtYTEyNS00ZDBlLTgxOTktYzY2OGU1ZDkxNDMw&hl=en

Standards for professional learning. (2011). Retrieved August 21, 2011, from the Learning forward website: http://www.learningforward.org/standards/standards.cfm
Willis, J. (2010). *Learning to love math: Teaching strategies that change student attitudes and get results*. ASCD.

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